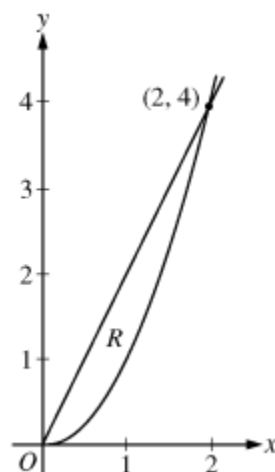


Area and Volume Classwork

1.

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

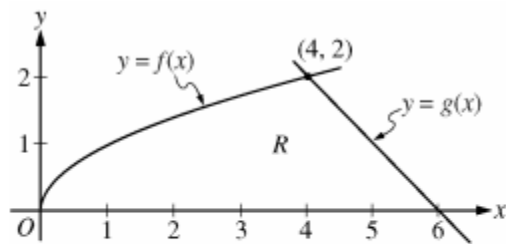
- Find the area of R .
- The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
- Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.



2.

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.

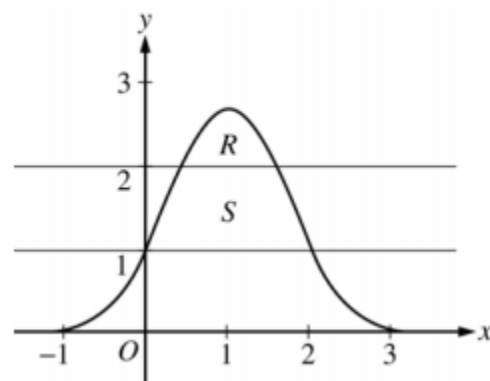
- Find the area of R .
- The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
- There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .



3.

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

- Find the area of R .
- Find the area of S .
- Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



4.

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.

